Neutrino scattering as a probe for the strange content of the nucleon *

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We consider different methods and observables which can be obtained by the measurement of neutrino scattering off nucleons and nuclei with the purpose of finding evidence for the strange form factors of the nucleon, which enter into structure of the nucleonic weak neutral current.

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1. Introduction

The contribution of the $s\bar{s}$ sea to the nucleon structure has been widely discussed in the recent past, especially in connection with the so-called "problem of the proton spin". It is related to the one nucleon matrix element of the axial quark current:

$$\langle p, s|\bar{q}\gamma^\alpha\gamma^5 q|p, s\rangle = 2Ms^\alpha g_A^q, \tag{1.1}$$

$q, \bar{q}$ being the quark fields and $|p, s\rangle$ the proton (momentum, spin) state vector.

Based on several assumptions, among which the use of the naive parton model (thus neglecting important QCD corrections) and SU(3) flavor symmetry, the constants g_A^u, g_A^d, g_A^s can be determined from:

1. QCD sum rule (of the polarized structure function)

$$\Gamma_1^p = \int_0^1 dx g_1^p(x) = \frac{1}{2}\left(\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\right)$$

2. the relation $g_A = g_A^u - g_A^d$
   with $g_A = 1.2573 \pm 0.0028$ obtained from neutron decay

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3. the relation $3F - D = g_A^u + g_A^d - 2g_A^s$
   where the constants F, D are measured from semileptonic decays of hyperons.

At face with the above mentioned theoretical uncertainties, it is highly desirable to find an alternative source of information for an independent determination of g_A^s [1].

In this perspective a very important tool is the measurement of neutral current (NC) neutrino cross sections

$$\nu_\mu(\overline{\nu}_\mu) + N \longrightarrow \nu_\mu(\overline{\nu}_\mu) + N \,, \tag{1.2}$$

together with the charge current (CC) processes:

$$\begin{aligned} \nu_\mu + n &\longrightarrow \mu^- + p \,, \\ \overline{\nu}_\mu + p &\longrightarrow \mu^+ + n \,. \end{aligned} \tag{1.3}$$

The neutral current involved in the above processes is:

$$J_\alpha^Z \equiv J_\alpha^{NC} = V_\alpha^3 + A_\alpha^3 - 2\sin^2\theta_W J_\alpha^{em} - \frac{1}{2}V_\alpha^s - \frac{1}{2}A_\alpha^s, \tag{1.4}$$

with $V_\alpha^3 = \bar{U}\gamma_\alpha U - \bar{D}\gamma_\alpha D$, $A_\alpha^3 = \bar{U}\gamma_\alpha\gamma_5 U - \bar{D}\gamma_\alpha\gamma_5 D$, $V_\alpha^s = \bar{S}\gamma_\alpha S$ and $A_\alpha^s = \bar{S}\gamma_\alpha\gamma_5 S$. The charge current reads, instead:

$$J_\alpha^W = V_{ud}\bar{U}\gamma_\alpha(1+\gamma_5)D. \tag{1.5}$$

The one-nucleon matrix elements of the above currents are usually expressed in terms of phenomenological form factors, which contain, in the NC sector, three isoscalar strange terms, G_E^s, G_M^s and G_A^s. It is our purpose to show how these form factors can be determined from ν $(\bar{\nu})$ scattering processes.

2. neutrino-nucleon (-nucleus) cross sections

2.1. Elastic NC ν–nucleon scattering

The differential cross section for the elastic NC ν-nucleon scattering reads [1]

$$\begin{aligned} \left(\frac{d\sigma}{dQ^2}\right)_{\nu(\overline{\nu})}^{NC} = \frac{G_F^2}{2\pi}\Bigg[\frac{1}{2}y^2(G_M^{NC})^2 + \left(1 - y - \frac{M}{2E}y\right)\frac{(G_E^{NC})^2 + \frac{E}{2M}y(G_M^{NC})^2}{1+\frac{E}{2M}y} \\ + \left(\frac{1}{2}y^2 + 1 - y + \frac{M}{2E}y\right)(G_A^{NC})^2 \pm 2y\left(1 - \frac{1}{2}y\right)G_M^{NC}G_A^{NC}\Bigg]\,, \end{aligned} \tag{2.1}$$

the + (-) sign referring to neutrinos (anti-neutrinos) respectively. In the above $y = \frac{p \cdot q}{p \cdot k} = \frac{Q^2}{2p \cdot k}$, E is the energy of the neutrino (antineutrino) in the laboratory system and the NC Sachs form factors can be expressed as:

$$G_E{}^{NC;p(n)}(Q^2) = \tag{2.2}$$
$$= \pm\frac{1}{2}\left\{G_E^p(Q^2) - G_E^n(Q^2)\right\} - 2\sin^2\theta_W G_E^{p(n)}(Q^2) - \frac{1}{2}G_E^s(Q^2)$$
$$G_M{}^{NC;p(n)}(Q^2) = \tag{2.3}$$
$$= \pm\frac{1}{2}\left\{G_M^p(Q^2) - G_M^n(Q^2)\right\} - 2\sin^2\theta_W G_M^{p(n)}(Q^2) - \frac{1}{2}G_M^s(Q^2)$$
$$G_A^{NC;p(n)}(Q^2) = \pm\frac{1}{2}G_A(Q^2) - \frac{1}{2}G_A^s(Q^2). \tag{2.4}$$

They clearly embody the isoscalar contribution of the electric (G_E^s), magnetic (G_M^s) and axial (G_A^s) strange form factors.

2.2. Quasi-Elastic CC ν–nucleon scattering

The differential cross section for the quasi-elastic CC ν-nucleon scattering reads [1]

$$\left(\frac{d\sigma}{dQ^2}\right)^{CC}_{\nu(\overline{\nu})} = \frac{G_F^2}{2\pi}\left[\frac{1}{2}y^2(G_M^{CC})^2 + \left(1 - y - \frac{M}{2E}y\right)\frac{(G_E^{CC})^2 + \frac{E}{2M}y(G_M^{CC})^2}{1 + \frac{E}{2M}y} + \right.$$
$$\left. + \left(\frac{1}{2}y^2 + 1 - y + \frac{M}{2E}y\right)(G_A)^2 \pm 2y\left(1 - \frac{1}{2}y\right)G_M^{CC}G_A\right]. \tag{2.5}$$

In the above the CC form factors appear.

2.3. Quasi-Elastic ν–nucleus scattering

Neutrino scattering is realized both on free and bound nucleons, often within the same experiment/detector: it is thus important to consider also ν–nucleus scattering processes and to accurately evaluate the effects of nuclear structure and dynamics.

The processes on a nucleus are, as before, of two types:

$$\nu_\mu(\overline{\nu}_\mu) + A \quad\longrightarrow \nu_\mu(\overline{\nu}_\mu) + N + (A-1) \qquad \text{NC process} \tag{2.6}$$
$$\nu_\mu(\overline{\nu}_\mu) + A \quad\longrightarrow \mu^-(\mu^+) + p(n) + (A-1) \qquad \text{CC process} \tag{2.7}$$

The approach one usually employs is the Impulse Approximation (IA), in which the neutrino interacts with a single nucleon in the nucleus; the latter can be described within the simplest available model, namely the (relativistic) Fermi Gas (RFG) or within a more refined relativistic shell model

(RSM) or taking into account initial state correlations (RPA and the like); an initial binding energy of the struck nucleon is customarily taken into account, also in RFG, though its effect is negligible for neutrino energies in the GeV range.

After weakly interacting with the neutrino, the nucleon ejected in the processes (2.6) and (2.7) can be treated either as a free one (PWIA) or as interacting with the residual nucleus (DWIA). Different approaches are available in order to deal with the distortion of the final nucleon [See more on this subject in Ref.[2, 3]].

In spite of its simplicity, the RFG model turns out to be useful, both in evaluating ratios of cross sections, where a large part of the nuclear effects fade away, and to get a feeling of how the various nucleonic form factors (including the strange ones) enter into the game. Indeed the RFG cross sections can be analytically evaluated and read:

$$\begin{aligned}\left(\frac{d^2\sigma}{dE_N d\Omega_N}\right)_{\nu(\overline{\nu})} &= \frac{G_F^2}{(2\pi)^2}\frac{3\mathcal{N}}{4\pi p_F^3}\frac{|\vec{p}_N|}{k_0}\int\frac{d^3k'}{k'_0}\frac{d^3p}{p_0}\delta\left(k_0-k'_0+p_0-E_N\right)\\ &\times\delta^{(3)}\left(\vec{k}-\vec{k}'+\vec{p}-\vec{p}_N\right)\theta(p_F-|\vec{p}\,|)\theta(|\vec{p}_N|-p_F)\\ &\times\left\{V_M(G_M^{NC})^2+V_{EM}\frac{(G_E^{NC})^2+\tau(G_M^{NC})^2}{1+\tau}+\right.\\ &\left.+\,V_A(G_A^{NC})^2\pm V_{AM}G_A^{NC}G_M^{NC}\right\},\end{aligned} \tag{2.8}$$

where $(\vec{p_N}, E_N)$ is the four momentum of the ejected nucleon, k (k') the momenta of the incoming (outgoing) neutrino and

$$\begin{aligned}&V_M = 2M^2\tau\,(k\cdot k')\\ &V_{EM} = 2\,(k\cdot p)\,(k'\cdot p) - M^2\,(k\cdot k')\\ &V_A = M^2\,(k\cdot k') + 2M^2\tau\,(k\cdot k') + 2\,(k\cdot p)\,(k'\cdot p)\\ &V_{AM} = 2\,(k\cdot k')\,(k\cdot p + k'\cdot p)\ ,\end{aligned} \tag{2.9}$$

the remaining symbols being self-explanatory.

The single differential cross sections then follow:

$$\left(\frac{d\sigma}{dT_N}\right)_{\nu(\overline{\nu})N} \equiv \left(\frac{d\sigma}{dE_N}\right)_{\nu(\overline{\nu})N} = \int d\Omega_N\left(\frac{d^2\sigma}{dE_N d\Omega_N}\right)_{\nu(\overline{\nu})N}\ , \tag{2.10}$$

T_N being the outgoing nucleon kinetic energy.

3. Interesting observables

As it has been suggested several times [1], the extraction of information on the strange form factors of the nucleon from ν-nucleon or ν-nucleus scattering cross section is better founded on the measurement of **ratios** of cross sections. This limits some of the experimental uncertainties as well as (in the case of ν-nucleus scattering) of the model dependence of the calculated cross sections.

Hence the following ratios have been proposed:

- NC over CC ratio (considered at Fermilab):

$$R_{NC/CC}(Q^2) = \left(d\sigma/dQ^2\right)_\nu^{NC} / \left(d\sigma/dQ^2\right)_\nu^{CC} \tag{3.1}$$

- Proton to neutron ratio (in quasi-elastic processes with emission of one nucleon) [4]:

$$R^\nu_{p/n}(Q^2) = \left(\frac{d\sigma}{dQ^2}\right)^{NC}_{(\nu,p)} / \left(\frac{d\sigma}{dQ^2}\right)^{NC}_{(\nu,n)} \tag{3.2}$$

- Neutrino-antineutrino Asymmetry [5]:

$$\mathcal{A}(Q^2) = \frac{\left(\frac{d\sigma}{dQ^2}\right)^{NC}_{\nu} - \left(\frac{d\sigma}{dQ^2}\right)^{NC}_{\overline{\nu}}}{\left(\frac{d\sigma}{dQ^2}\right)^{CC}_{\nu} - \left(\frac{d\sigma}{dQ^2}\right)^{CC}_{\overline{\nu}}} \tag{3.3}$$

Since mono-energetic neutrinos are not available, in the above combinations it is customary to employ flux averaged neutrino cross sections:

$$\langle\frac{d\sigma}{dQ^2}\rangle^{NC}_{\nu(\overline{\nu})} = \frac{\int dE_{\nu(\overline{\nu})} \left(d\sigma/dQ^2\right)^{NC}_{\nu(\overline{\nu})} \Phi_{\nu(\overline{\nu})}\left(E_{\nu(\overline{\nu})}\right)}{\int dE_{\nu(\overline{\nu})} \Phi_{\nu(\overline{\nu})}\left(E_{\nu(\overline{\nu})}\right)} \,, \tag{3.4}$$

the flux $\Phi_{\nu(\overline{\nu})}(E)$ being provided by the various experiments.

3.1. The $\nu - \bar{\nu}$ asymmetry

The neutrino-antineutrino asymmetry in $\nu(\bar{\nu})$-nucleon elastic scattering explicitly reads [5]:

$$\mathcal{A}_{p(n)} = \frac{1}{4}\left(\pm 1 - \frac{G_A^s}{G_A}\right)\left(\pm 1 - 2\sin^2\theta_W \frac{{G_M}^{p(n)}}{G_M^3} - \frac{1}{2}\frac{G_M^s}{G_M^3}\right) . \tag{3.5}$$

Thus, in the asymmetry $\mathcal{A}$ the strange axial and vector form factors enter in the form of ratios, G_A^s/G_A and G_M^s/G_M^3. Taking into account only terms which linearly depend on the strange form factors one gets:

$$\mathcal{A}_{p(n)} = \mathcal{A}^0_{p(n)} \mp \frac{1}{8}\frac{G_M^s}{G_M^3} \mp \frac{G_A^s}{G_A}\mathcal{A}^0_{p(n)} \tag{3.6}$$

with $\mathcal{A}^0_{p(n)} = \frac{1}{4}\left(1 \mp 2\sin^2\theta_W \frac{G_M{}^{p(n)}}{G^3_M}\right)$.

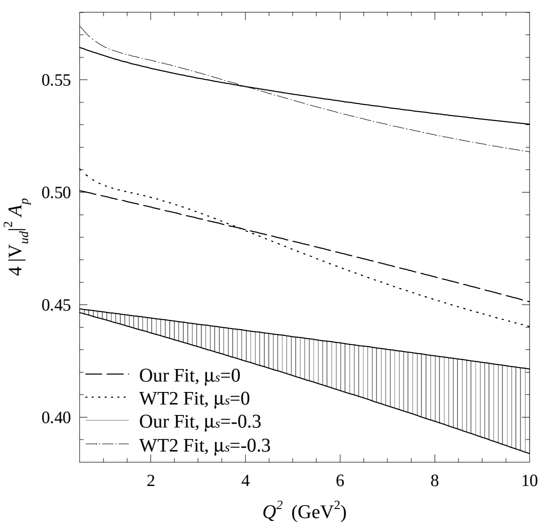

Fig. 1. Plot of $4|V_{ud}|^2\mathcal{A}_p$ as a function of Q^2. In the upper lines $g_A^s = -0.15$.

The quantity (3.6) is plotted in Fig.1, where the shaded area (corresponding to the uncertainty in the magnetic form factors) is the result obtained without strange form factors. Should the latter be different from zero, their effect would be clearly visible.

4. The BNL - 734 experiment

In 1987 the 734 experiment at BNL measured the following ratios [6]:

$$R_\nu = \frac{\langle\sigma\rangle_{(\nu p\to\nu p)}}{\langle\sigma\rangle_{(\nu n\to\mu^- p)}} = 0.153 \pm 0.007 \pm 0.017 \tag{4.1}$$

$$R_{\overline{\nu}} = \frac{\langle\sigma\rangle_{(\overline{\nu} p\to\overline{\nu} p)}}{\langle\sigma\rangle_{(\overline{\nu} p\to\mu^+ n)}} = 0.218 \pm 0.012 \pm 0.023 \tag{4.2}$$

$$R = \frac{\langle\sigma\rangle_{(\overline{\nu} p\to\overline{\nu} p)}}{\langle\sigma\rangle_{(\nu p\to\nu p)}} = 0.302 \pm 0.019 \pm 0.037\ , \tag{4.3}$$

where $\langle\sigma\rangle_{\nu(\bar{\nu})}$ is a total cross section integrated over the incident neutrino (antineutrino) energy and weighted by the $\nu(\bar{\nu})$ flux. The first error is statistical and the second is the systematic one.

In terms of these ratios, the "integrated" asymmetry reads:

$$\langle\mathcal{A}_p\rangle = \frac{R_\nu(1-R)}{1-RR_\nu/R_{\overline{\nu}}} \tag{4.4}$$

and from the experimental data we found [7]

$$\langle\mathcal{A}_p\rangle = 0.136 \pm 0.008(\mathrm{stat}) \pm 0.019(\mathrm{syst}), \tag{4.5}$$

which is the only existing measurement of the neutrino asymmetry. The RFG ratios (4.1)–(4.3) are plotted in Fig.2, together with the asymmetry, as functions of the magnetic strangeness and for different choices of parameters entering into the axial (g_A^s) and electric (ρ_s) strange form factors [for a parameterization of the latter see Ref.[1]]. The shaded areas correspond to the experimental band.

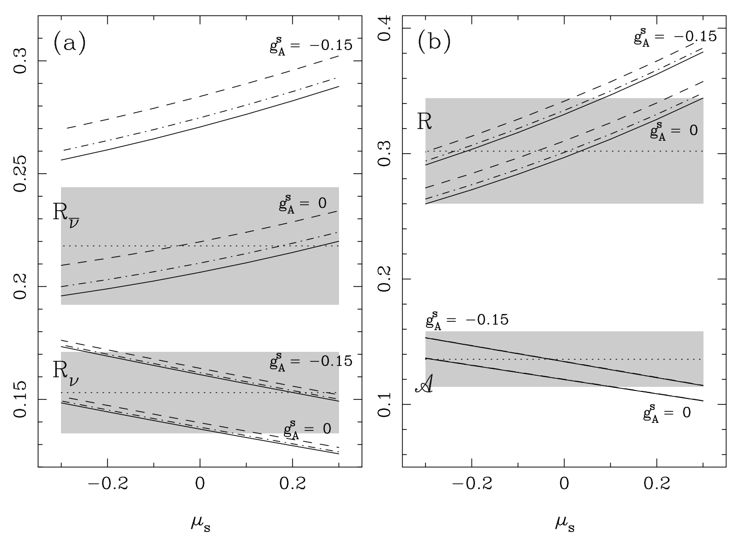

Fig. 2. R_ν and $R_{\bar{\nu}}$, R and $\langle\mathcal{A}_p\rangle$, for $g_A^s = 0$ and $g_A^s = -0.15$. Three choices of ρ_s are shown: $\rho_s = 0$ (solid),$\rho_s = -2$ (dot–dashed) and $\rho_s = +2$ (dashed).

The figure clearly shows the sensitivity of the various ratios to the strange form factors: notice that the quantities (4.1) and (4.2) are ratios of

NC over CC cross sections. Unfortunately the error band of the BNL-734 experiment are too large to draw definitive conclusions, but they certainly do not exclude relevant contribution of the strange sea to the nucleon structure.

5. Conclusions and future perspectives

The experiments of ν-proton NC and CC scattering are highly interesting for the determination of $\Delta s \equiv g_A^s$. Among the various proposed observables, the ration of NC and CC elastic νp scattering cross sections is being considered in several proposal at Fermilab (Minerva, FINESSE).

This quantity will be sensitive to g_A^s, but possibly not much affected by the cutoff mass of the axial form factors (assuming, for simplicity, the same dipole form in the strange form factors as in the ordinary ones). Also the e.m. form factors do not sensibly affect the NC/CC ratio.

One of the major uncertainties in the extraction of g_A^s from ν-N scattering stems from the unavoidable interference between the axial and the magnetic (μ_s) strange form factors. Neutrino scattering alone can only determine a linear combination of both; hence it is highly desirable to obtain a complementary information on μ_s from a different source. This is the case of parity violating (PV) scattering of polarized electrons from protons [1], a process which is mainly sensitive to the vector NC form factors. A recent determination of μ_s has been obtained by the HAPPEX experiment at TJLab [8], also combined with previous measurements at BATES [9]. Hence one can hope to get the relevant information on g_A^s from the future/planned neutrino experiments, which appear to be a unique tool for an unambiguous determination of Δs.

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